SOCIAL ENHANCEMENT OF WORLD POPULATION GROWTH

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Empirical studies show that past and present data on the world population fit exceptionally well to a rate expression which depends on the square of population rather than the first power as is usually assumed. It is concluded that for many years, perhaps as many as a million, the growth rate has been enhanced by the social support of the surrounding populace, and has never followed simple exponential behavior.

INTRODUCTION

That the human population grows exponentially has been a widely accepted concept at least since the time of Malthus. This idea has been elaborated in works of great mathematical sophistication (1). It is only necessary to plot the data on world population on a suitable (logarithmic) plot as in Fig. 1 to see that exponential behavior does not come near to describing the facts. On such a plot the data would fall on a straight line if the growth had been exponential. Clearly it does not.

The exponential differential equation $dN/dt = \beta N$ (1)

has as its solution the algebraic form $N = N_0 \exp(\beta_t)$ where β is the rate of growth. If it is written in the equivalent form $d(\log N)/dt = \beta$, one sees at once that the slope of a logarithmic curve such as was plotted in Fig. 1 gives the growth rate constant at each epoch at which it is measured. If such measurements are made and plotted against their corresponding epochs they are not especially revealing, but if plotted against the populations prevailing at those epochs, the rather startling fact emerges that growth *rate* has been quite accurately *linearly* proportional to population over very long periods, i.e., $\beta \propto N$. Although this logical process was followed in the development of this paper, a much more precise test is offered here.

Since $\beta = \alpha N$, where α is a constant, we may substitute in Eq. 1 to obtain an equation well known in chemistry and physics where it governs binary reactions:

$$dN/dt = \alpha N^2 \qquad (2)$$

$$l/N = \alpha (t_\infty - t) \qquad (3)$$

If now the solution of this binary reaction equation is used as a basis for plotting the data, with reciprocal populations as ordinates as in Fig. 2, an extremely good fit to a straight line is obtained, confirming the argument that led to the binary equation.

The high quality of the agreement between predictions from the binary law and the actual data over the contemporary epoch +1600 to +1978 leads one to consider applying it to past and future times where the data are either less well known or totally lacking. However, to make such extrapolations we need to examine the factors underlying a binary equation for population growth.

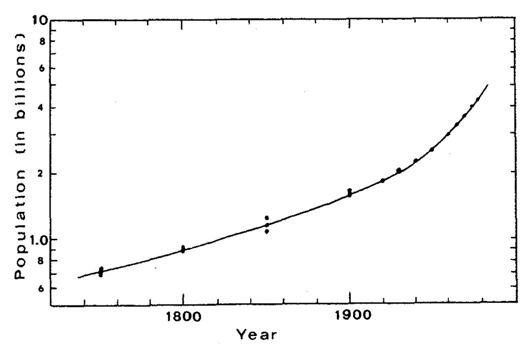

FIGURE 1. Logarithmic plot of world population +1750 to +1978, showing that growth has not been exponential at any time. The curve is a simple connection of the points. For exponential growth is should be a straight line on this plot.

A MODIFIED POPULATION THEORY

The population growth rate is the difference between the birth and death rate. Each of these rates must have a linear dependence on population since both (particularly the death rate) have a spontaneous element. From the success of the binary equation we can deduce that added to the linear term in one or both processes there must also be a quadratic term implying that merely the presence of more people alters the rate. That is, in general,

$$dN/dt = \beta_1 N + \beta_2 N^2 - \delta_1 N - \delta_2 N^2 . \qquad (4)$$

Now β_1 and δ_1, the spontaneous rates, must be positive quantities. However, β_2 and δ_2, the socially enhanced rates, may be either positive or negative, i.e., on the basis of social support people may feel conditions to be favorable to unfavorable for extra births, and equally, society may act to increase or reduce longevity. The empirical observation for the period +1600 to +1978 is that β_2 has turned positive, and δ_2 has been negative, so that $\beta_2 — \delta_2$, which was denoted by α in the binary equation, is positive. But more striking is the fact that to achieve the binary fit, β_1 has been equal to δ_1, over this period within the undetectable difference of 2×10^{-6}, while the values of β_1 and δ_1 have been around 2×10^{-2}. The binary equation must not be confused with the so-called logistic equation, in which $\beta_1 — \delta_1$ is positive, not zero, and $\beta_2 — \delta_2$ is negative. This has considerably different solutions and consequences.

To gain a real appreciation of the difference in predictive power between these two laws, it is interesting to compare the predictions that Malthus himself could have made from the data available on the period +1600 to +1800. These are given in Table 1. Although further study of the whole data set shows that the extraordinary excellence of the binary predictions for the second half of the 20th century is somewhat fortuitous, there is no way that the total failure of the exponential prediction can be rationalized except by readjusting the growth rate each year, which in effect is abandoning it, and unconsciously substituting the binary law. On the other hand, whatever range of data between +1600 and +1978 is chosen as a basis for fitting, predictions from the binary law for the rest of the range never miss any point by more than 10%, while the overall fit of the data in Fig. 2 to the theoretical curve is ± 2.3%, which is certainly as good as the early data. In fact, if the study is restricted to the rather more accurate data since 1920, the fit is ± 0.59%.

The above analysis is relevant to any attempt to extrapolate the binary growth equation to the future, because it suggests a true population explosion, i.e., an infinite value (singularity) achieved at some finite time. This is in contrast to exponential growth, where infinite population is attained, but at infinite time. As of +1978 the indicated date of this predicted singularity is the astoundingly imminent year +2025 (4).

Certainly singularity cannot transpire, if only because female fertility places a fundamental limit on growth rate. Nevertheless there is nothing in the trend of data for the past ten years to hint of any change in the growth law. Contrary to conventional opinion, average female fertility has apparently been rising to maintain the socially enhanced growth process. From +1400

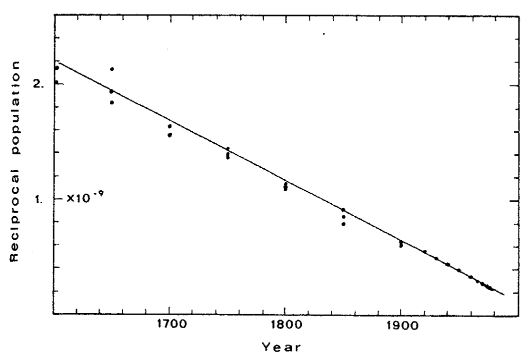

FIGURE 2. Reciprocal world population estimated +1600 to +1978 plotted against date. Straight line is weighted least squares fit to data with highest weights to most recent data points.

TABLE 1. *Retrospective 1800 Population Predictions (billions)*

Year	1850	1900	1920	1940	1960	1970	1978
Actual (2,3)	1.17	1.62	1.81	2.25	2.986	3.610	4.258
Exponential	1.17	1.43	1.58	1.73	1.91	2.00	2.08
Binary	1.14	1.56	1.90	2.35	3.08	3.64	4.26

to +1900 fertility averaged about 0.08 liveborn child per year per female of childbearing age. Since +1900 it has risen steadily until it is now about 0.11. Much of the increase has no doubt arisen from a worldwide reduction in infant mortality owing to better medical care, but this is precisely the meaning and one of the causes of the binary law.

For the binary law to hold to the year +2000 female fertility would need to increase to about 0.22. This is not beyond reason, since a rate of .275 was present in the United States in 1800. Whether this takes place remains to be seen, but the absolute limit is around 1.0, and even this would need to be reached by +2015 to maintain binary growth, so we may reasonably expect a substantial decrease of the growth rate, probably sometime before the year +2000.

The United Nations Statistical Yearbook (3), using essentially an exponential extrapolation from the +1970 to +1980 base, expects 5.3 and 6.2 billion people for +1990 and +2000 respectively. The binary law predicts 5.7 and 8.0 billion. That prediction for +1990 may well be excellent, but control of female fertility will probably hold the +2000 population to about 7.7 billion. 7.7 and 8.0 billions may thus, barring catastrophe or a genuine world awakening, be regarded as lower and upper limits on the population for +2000, considering the inexorable trend of the present data.

EXTENSION TO PREHISTORY

The more rewarding applications of the binary law may come from extrapolations to the past, where new social and anthropological insights could follow. Such extrapolations will certainly be controversial, but in the end the binary law seems likely to offer another yardstick to probe the past. Reasonably good estimates of world

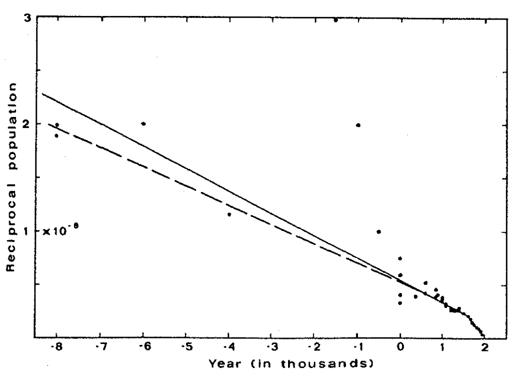

FIGURE 3. Reciprocal world population estimates back to −8000 plotted against data. Solid line is least squares fit to all data −8000 to +1600. Dashed line is extrapolated fit from data between 0 and +1600. Note abrupt increase in growth rate at +1600. Exponentially based estimates of McEvedy and Jones (stars) are not included in the fit.

TABLE 5. *Numerical Comparisons (Billions)*

	Source								
	U.N. (3)	Durand (2)	Carr-Saunders (10)	Willcox (11)	Clark (6)	McEvedy & Jones (8)	Deevey (5)	Bennett (12)	Binary law
1978	4.258								4.224
1976	4.044								4.051
1974	3.890								3.891
1970	3.610								3.606
1940		2.25							2.33
1920		1.81							1.88
1900		1.65	1.61	1.57		1.63			1.58
1800			0.90	0.919	0.890	0.900			0.877
1700					0.641	0.612			0.606
1600					0.498			0.486	0.480
1400						0.350		0.373	0.399
1200					0.384	0.340		0.348	0.342
1000					0.280	0.265		0.275	0.299
600					0.237	0.190			0.239
0	0.2 to 0.3	0.30				0.170	0.133		0.184
−4000						0.007	0.0865		0.0723
−6000		0.05				.0045			0.0555
−8000							0.0532		0.0450

population have been made back to about -8000. When these are added to a re-scaled inverse population plot as in Fig. 3, the first unexpected social fact emerges. There was a marked and abrupt increase in population growth rate around the year +1600. Exactly when it began is obscured by the oscillation precipitated by the great plagues but it was certainly after +1200. After this change in growth rate, the curve fit to the data is again striking, and as good as the data permits. It would be good even if extrapolated from the range 0 to +1600 (dashed curve in Fig. 3). The two sets of fitting constants are given in Table 2.

The point symbols used on the graphs are unavoidably large and do not show the quality of the agreement. Actual numerical comparisons are given at the end in Table 5 for some unselected points.

Deevey (5), Clark (6), Keyfitz (7), and McEvedy and Jones (8) have given population estimates for even more remote times. If the data fit between +1600 and -8000 is extrapolated, predictions to match their estimates can be made, and are given in Table 3 and 4. In Table 3 population predictions at specific dates are compared. In Table 4, dates are predicted to match a date range estimate for a specific population value.

The binary law may be regarded as a harmonizing device which both gains its strength by its agreement with these estimates, and in turn strengthens them by showing their mutual consistency even over such a long time span as 2 million years.

The only estimates known to the author which disagree seriously are the recent ones by McEvedy and Jones for the period -10^3 back to -10^6. They state that modern anthropological opinion is that a long period of stationary population enduring from -10^6 to -3×10^4 was succeeded by a period of rapid growth. It is evident from their numbers that they used an exponential model to calculate the data for this hypothetical growth period. Their numbers not only disagree with binary predictions, but with the half-dozen or so other estimates which have been made by other investigators. It seems rather likely that the theory of a static population followed by exponential growth has arisen because it is not possible to fit a reasonable exponential curve to the span of time and population involved in

TABLE 2. *Fitting Constants for Population Data.*

Epoch	α	t_∞	Error
+1978 to +1600	5.079×10^{-12}	2024.6	± 2.3%
+1600 to −8000	2.099×10^{-12}	2591.5	± 9.3%

TABLE 3. *Populations in Prehistory*

Date	Population Prediction	Population Estimate
−23,000	1.9×10^7	3.3×10^7 (Deevey)
-3×10^5	1.6×10^6	1 to 2×10^6 (Deevey)
-10^6	4.8×10^5	1 to 5×10^5 (Deevey)
-2.5×10^6	2.0×10^5	0.7 to 7×10^5 (McEvedy - Jones)

TABLE 4. *Populations in Prehistory*

Population	Date Prediction	Date Estimate
1.0×10^7	-5.5×10^4	−1.0 to -10×10^4 (Keyfitz)
1.5×10^6	-3.5×10^5	−1.5 to -15×10^5 (Clark)

prehistory (9). Binary growth does permit such a continuous fit.

It would be naive to assume that true population numbers followed the binary growth curve rigidly throughout all history. Certainly, as was clearly shown during the great plagues, population must at best have oscillated around binary growth as a kind of mean trend. Moreover, world population studies are strongly weighted toward Oriental populations, and the existence of the binary law must heavily reflect Oriental cultural patterns. Patterns of growth in other places can clearly deviate from it. Thus, the growth of population in the United States was purely exponential until 1840, but after that time can be shown to contain a binary component with the same world coefficient.

The binary growth equation cannot be extrapolated beyond about -1.5×10^6, the advent of *Homo Erectus*. No choice of constants can describe the period between then and the appearance of hominids in -5.5×10^6. During that interval only an exponential growth process, albeit at the very low net rate of 4×10^{-6} persons per person per year, can make the connection in finite time from unit population to a population even as small as 10^5. The appearance of exponential growth at this time seems consistent with an animal character for Australopithecene man. Exponential growth is a natural characteristic of animal populations. Only rational man would institute practices leading to growth rates which depended not only on fertility, but on cooperation from his own species.

SUMMARY AND CONCLUSIONS

In summary, binary growth laws seem to have dominated mankind over recorded history, a fact consistent with the principal difference between animals and mankind — intellectual powers. A single important change occurred in the binary rate coefficient in about +1600. Considering the probable basis of the binary law, this may relate to the development of increasingly sophisticated world-wide public health measures following the great plagues. The fact that the binary law can be extrapolated to the advent of *Homo Erectus* suggests that he was the first to see the advantages of cooperation in propagation of his species, and this supported his ascendancy. Finally, the binary process cannot persist in our modern world, but must be replaced by something no more severe than exponential growth at a rate below maximal female fertility. Ominous as this sounds, it will be less catastrophic than the binary process under which world population presently grows.

The principal conclusion to be drawn is that it is possible with a simple law and very limited number of constants to subsume both the actual data and the estimates of many workers over an enormous span of history. Such a result can hardly be accidental, and deserves serious consideration and discussion whether the explanation offered here is correct or not. It could be the source of many new insights.

REFERENCES

1. H. H. POLLARD, *Mathematical Models for the Growth of Human Population*, Cambridge, 1973.
2. J. D. DURAND, Proc. Am. Phil. Soc. 111: 133 (1967).
3. UNITED NATIONS STATISTICAL YEARBOOK 30: 2 (1978).
4. The existence of this singularity was pointed out twenty years ago by H. V. FOERSTER, P. M. MORA, and L. W. AMIOT, Science 132: 1291 (1960). Examining data from 0 to + 1960, they noted that such data could be fitted to the equation $N = A(t_\infty - t)^{-k}$ and trended toward a value of t_∞ of +2027. The mathematical and anthropological basis for the value of k being exactly unity was not discussed, and their observation has largely passed unnoticed. The present article is an independent rediscovery of what seems to be an important idea: world population growth is not exponential.
5. E. S. DEEVEY, Sci. Am. 203 (3): 195 (1960).
6. C. CLARK, *Population Growth and Land Use*, MacMillan, New York, 1968.
7. N. KEYFITZ, Demography 3: 581 (1966).
8. C. McEVEDY and R. JONES, *Atlas of World Population History,* Wiley, New York, 1977.
9. R. L. CARNIERO and D. HILSE, Am. Anthropol. 68: 177 (1977) have shown that exponential growth calculations can never fit growth in the -10^3 to -10^6 epoch, and conclude therefore that population must have been nearly constant.
10. A. M. CARR-SAUNDERS, *World Population*, Clarendon Press, Oxford, 1936.
11. W. F. WILLCOX, Increase in the Population of the Earth and of the Continents since 1650. *In* W. F. WILLCOX (ed.), *International Migrations*, Vol. II, Natl. Bureau of Economic Research, New York, 1931, pp. 33-82.
12. MERRILL K. BENNETT, *The World's Food.* Harper, New York, 1954.